EVALUATION OF THE RADIATION CONTRIBUTION IN SHORT-DURATION MEASUREMENTS OF THERMAL CONDUCTIVITY BY A NONSTATIONARY METHOD OF HOT FILAMENT

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An analytical expression for evaluation of the effect of radiation on results of short-duration measurements of the molecular thermal conductivity of semitransparent media is obtained based on the solution of the problem of radiative-conductive heat transfer (RCHT).

The possibility of almost completely eliminating radiative heat transfer is a qualitatively new special feature of the method of short-duration measurements of thermophysical characteristics in which, in particular, a linear heat source is used (a nonstationary method of hot filament). The organization of the experiment under these conditions allows one to unambiguously classify the characteristics of the transfer as molecular [1].

Improvement of the accuracy of measurements requires more specific quantitative evaluations of the effect of radiation on the results of measuring the characteristics of heat transfer.

In [2], the problem of RCHT for the case of nonstationary heating of the linear source (in a "gray" approximation of the studied medium, with regard to the second cylindrical boundary) was considered numerically. However, calculation even for one set of parameters is computer-time consuming and it is difficult to make a rapid analysis of the radiation contribution under other conditions or to generalize results based on a numerical calculation.

An analytical solution of the considered problem of RCHT obtained under a number of simplifying assumptions is given in [3]. In particular, it is assumed that the main term that determines radiative heat transfer is the radiation of a volume element. Assuming the linear heat source to be infinitely thin, the authors used a boundary condition that fails badly to allow for radiative transfer.

The mentioned approach casts some doubt, since it is clear from physical considerations that radiation emitted by the heat source can, in the general case, exert a larger effect on the process of RCHT than the radiation emitted by the volume of the studied semitransparent medium.

We consider the corresponding problem in a more general formulation.

We write an equation of heat transfer in the form

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) - \frac{\operatorname{div} \vec{q}^{\,\mathrm{rad}}(r, t)}{\rho c} \tag{1}$$

with the boundary condition

$$\left(\frac{\partial T}{\partial r}\right)_{r=r_0} = -\frac{q_{\mathrm{lin}}}{2\pi r_0 \lambda} + \frac{q^{\mathrm{rad}}(r_0, t)}{\lambda} \tag{2}$$

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and the initial condition

$$(T)_{t=0} = T_0 .$$

Using the expression for the radiation flux q^{rad} for the case of cylindrical symmetry of the temperature field and the approximation of an optically thin layer [4], we obtain

$$\operatorname{div} \vec{q}^{\,\text{rad}} (r, t) = \frac{q^{\text{rad}}}{r} + \frac{\partial q^{\text{rad}}}{\partial r} \approx 16 \kappa n^2 \sigma T_0^3 (T (r, t) - T_0) ,$$

$$q^{\text{rad}} (r_0, t) = 4 (1 - \rho^{\text{s}}) n^2 \sigma T_0^3 (T (r_0, t) - T_0) .$$

Thus, if, in measuring, the condition of an optically thin layer

$$\text{Kn} = \kappa \sqrt{at} << 1 ,$$

is realized, then Eqs. (1) and (2) can be represented in the form

$$\frac{\partial T}{\partial t} = a \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - B (T - T_0) , \tag{3}$$

$$\left(\frac{\partial T}{\partial r} \right)_{r=r_0} = - \frac{q_{\text{lin}}}{2 \pi \lambda r_0} + \frac{C}{r_0} ((T)_{r=r_0} - T_0) , \tag{4}$$

where $B = 16 \kappa n^2 \sigma T_0^3 / (\rho c)$; $C = 4(1 - \rho^{\text{s}}) n^2 \sigma T_0^3 r_0 / \lambda$.

We solve (3) and (4) by the method of iterations.

We consider the corresponding expressions in the zeroth approximation (radiative heat transfer is absent):

$$\frac{\partial T^0}{\partial t} = a \left(\frac{\partial^2 T^0}{\partial r^2} + \frac{1}{r} \frac{\partial T^0}{\partial r} \right) , \quad r > r_0 ;$$

$$\left(\frac{\partial T^0}{\partial r} \right)_{r \to r_0} = - \frac{q_{\text{lin}}}{2 \pi \lambda r_0} ; \tag{5}$$

$$(T^0)_{t=0} = T_0 .$$

The solution of (5) is given in [5, p. 337]:

$$T^0 (r, t) - T_0 = \frac{q_{\text{lin}}}{\pi^2 \lambda} \int_0^\infty (1 - \exp (- \text{Fo}\, u^2)) \frac{Y_0 \left(u \frac{r}{r_0} \right) J_1 (u) - J_0 \left(u \frac{r}{r_0} \right) Y_1 (u)}{\varphi (u)} \frac{du}{u^3} , \tag{6}$$

$$T^0 (r_0, t) - T_0 = \frac{2 q_{\text{lin}}}{\pi^3 \lambda} \int_0^\infty \frac{(1 - \exp (- \text{Fo}\, u^2))}{\varphi (u)} \frac{du}{u^3} = \frac{2 q_{\text{lin}}}{\pi^3 \lambda} \mathcal{F}_{\text{cond}} (\text{Fo}) . \tag{7}$$

Here

$$\varphi(u) = J_1^2(u) + Y_1^2(u), \quad \mathcal{F}_{\text{cond}}(\text{Fo}) = \int_0^\infty \frac{(1 - \exp(-\text{Fo}\, u^2))}{\varphi(u)\, u^3} du .$$

We immediately note that

$$L[T^0(r, t) - T_0] = \frac{q_{\text{lin}} \sqrt{a}\, K_0(\beta r)}{2\pi r_0 \lambda s^{3/2} K_1(\beta r_0)}, \tag{8}$$

$$L[T^0(r_0, t) - T_0] = \frac{q_{\text{lin}} \sqrt{a}\, K_0(\beta r_0)}{2\pi r_0 \lambda s^{3/2} K_1(\beta r_0)}. \tag{9}$$

Here $\beta = \sqrt{s/\alpha}$

Assuming that the contribution of radiation is small compared to that of conduction, we substitute (6) and (7) into radiative terms (3) and (4), respectively. We obtain first-approximation equations in the form

$$\frac{\partial T^1}{\partial t} = a\left(\frac{\partial^2 T^1}{\partial r^2} + \frac{1}{r}\frac{\partial T^1}{\partial r}\right) - B(T^0(r, t) - T_0), \quad r > r_0;$$

$$\frac{\partial T^1}{\partial r} = -\frac{q_{\text{lin}}}{2\pi r_0 \lambda} + \frac{C}{r_0}(T^0(r_0, t) - T_0), \quad r = r_0 .$$

Transforming these equations according to Laplace, with account for (8) and (9) we have

$$\theta'' + \frac{1}{r}\theta' - \frac{s}{a}\theta = -\frac{T_0}{a} + D\frac{K_0(\beta r)}{s^{3/2} K_1(\beta r_0)}, \quad r > r_0; \tag{10}$$

$$\theta' = -\frac{q_{\text{lin}}}{2\pi\lambda r_0 s} + A\frac{K_0(\beta r_0)}{s^{3/2} K_1(\beta r_0)}, \quad r = r_0, \tag{11}$$

where $A = Cq_{\text{lin}}\sqrt{a}/(2\pi\lambda r_0^2)$; $D = Bq_{\text{lin}}/(2\pi\lambda r_0\sqrt{a})$.

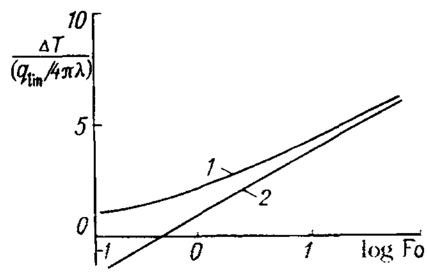
The general solution of the inhomogeneous Bessel equation (10) has the form [6]

$$\theta = C_* K_0(\beta r) + \frac{T_0}{s} + D\int_r^\infty \frac{K_0(\beta\tau)\,\tau}{s^{3/2} K_0(\beta r_0)}[K_0(\beta r) I_0(\beta\tau) - I_0(\beta r) K_0(\beta\tau)]\, d\tau, \tag{12}$$

and we find C_* by differentiating (12) and substituting $\partial\theta/\partial r$ into (11). Using properties of the Bessel function, we have

$$\theta|_{r=r_0} = \frac{T_0}{s} + \frac{q_{\text{lin}} K_0(\beta r_0)}{2\pi\lambda r_0 s K_1(\beta r_0)\beta} - \frac{AK_0^2(\beta r_0)}{s^{3/2} K_1^2(\beta r_0)\beta} - \frac{D}{r_0}\int_{r_0}^\infty \frac{\tau K_0^2(\beta\tau)}{s^{3/2}\beta K_1^2(\beta r_0)} d\tau . \tag{13}$$

Omitting intermediate calculations that are associated with the use of an inverse Laplace transform, we write the result for the temperature of the linear source [7]



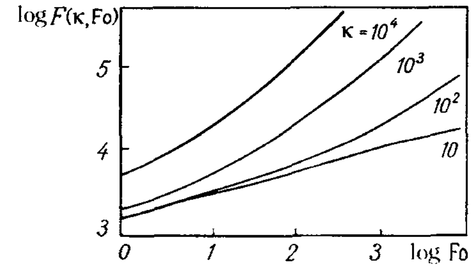


Fig. 1. Dependence of the temperature of a linear source on log Fo: 1) by results of numerical calculation by formula (16); 2) solution used in the literature (formula (15)).

Fig. 2. Graph of the function $F(\kappa, \mathrm{Fo})$, $r_0 = 5\cdot 10^{-6}$ m, F, m^{-1}.

$$T^1(r_0, t) = T_0 + \frac{2q_{\mathrm{lin}}}{\pi^3\lambda}\mathcal{F}_{\mathrm{cond}}(\mathrm{Fo}) + \frac{4n^2\sigma T_0^3 q_{\mathrm{lin}} r_0^2}{\lambda^2\pi}\mathcal{F}(\kappa, \mathrm{Fo}), \tag{14}$$

where

$$\mathcal{F}(\kappa, \mathrm{Fo}) = \left(\kappa + \frac{\rho^s - 1}{2r_0}\right)\frac{16}{\pi^3}G(\mathrm{Fo}) - \kappa\,\mathrm{Fo};$$

$$G(\mathrm{Fo}) = \int_0^\infty\int_0^\infty \frac{R(u, v, \mathrm{Fo})\,du dv}{(uv)^3\,\varphi(u)\,\varphi(v)};$$

$$R(u, v, \mathrm{Fo}) = \frac{\exp(-\mathrm{Fo}\,u^2)\,v^2}{u^2 - v^2} + \frac{\exp(-\mathrm{Fo}\,v^2)\,u^2}{v^2 - u^2} + 1.$$

The second term of expression (14) reflects conductive heat transfer, the third term – radiative transfer. We consider the conductive component in the source temperature in more detail. The existing expression for the temperature of a linear source in the form [5]

$$\Delta T_{\mathrm{ln}} = T_{\mathrm{ln}}(r_0, t) - T_0 = \frac{q_{\mathrm{lin}}}{4\pi\lambda}\ln\frac{4at}{r_0^2\exp(C_{\mathrm{E}})} \tag{15}$$

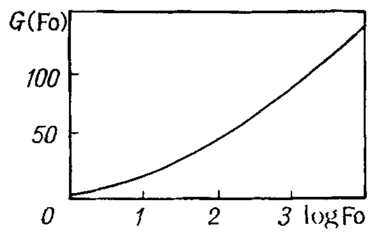
corresponds to a source of infinitely small radius and therefore is approximate as compared to the conductive component from (14)

$$\Delta T_{\mathrm{cond}} = \frac{2q_{\mathrm{lin}}}{\pi^3\lambda}\mathcal{F}_{\mathrm{cond}}(\mathrm{Fo}). \tag{16}$$

The dependence of the temperature of the linear source (16) on log Fo obtained by numerical integration is shown in Fig. 1 (curve 1). The corresponding dependence for ΔT_{ln} in the coordinates adopted is shown by straight line 2 and is the asymptote of the graph of the function ΔT_{cond} when log Fo $\to \infty$. They virtually coincide when log Fo > 2.

Having rewritten (14) in the form

$$T^1(r_0, t) - T_0 = \Delta T_{\mathrm{cond}}(r_0, t) + \delta T_{\mathrm{rad}}(r_0, t),$$



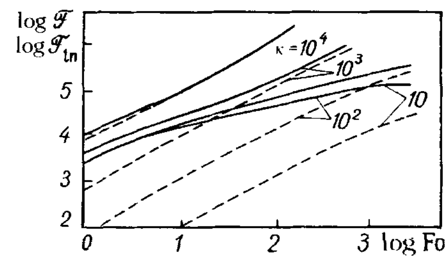


Fig. 3. Graph of the function $G(\mathrm{Fo})$.

Fig. 4. Comparison of the functions $\mathcal{F}(\kappa, \mathrm{Fo})$ and $\mathcal{F}_{\ln}(\kappa, \mathrm{Fo})$: solid lines, $\log \mathcal{F}(\kappa, \mathrm{Fo})$ (the source radius $r_0 = 5 \cdot 10^{-6}$ m); dashed lines, $\log \mathcal{F}_{\ln}(\kappa, \mathrm{Fo})$. $\mathcal{F}$, $\mathcal{F}_{\ln}$, m^{-1}.

we have for the relative contribution of radiation

$$\frac{\delta T_{\mathrm{rad}}}{T^{1} - T_0} \approx \frac{\delta T_{\mathrm{rad}}}{\Delta T_{\mathrm{cond}}} = \frac{2n^2 \sigma T_0^3 r_0^2 \pi^2 \, \mathcal{F}(\kappa, \mathrm{Fo})}{\lambda \, \mathcal{F}_{\mathrm{cond}}(\mathrm{Fo})} = \frac{2n^2 \sigma T_0^3 r_0^2 \pi^2}{\lambda} F(\kappa, \mathrm{Fo}),$$

where

$$F(\kappa, \mathrm{Fo}) = \frac{\mathcal{F}(\kappa, \mathrm{Fo})}{\mathcal{F}_{\mathrm{cond}}(\mathrm{Fo})}. \tag{17}$$

The effect of radiation on the measurement of the molecular thermal conductivity of semitransparent substances can be analyzed using graphs of the function $F(\kappa, \mathrm{Fo})$ (Fig. 2). Each curve of a graph is the dependence of $F(\kappa, \mathrm{Fo})$ on the Fourier number for a fixed value of the coefficient of absorption κ($\rho^{s} = 0.94$, $r_0 = 5 \cdot 10^{-6}$ m).

If it is necessary to calculate with other values of r_0 and ρ^{s}, then a graph of the function $G(\mathrm{Fo})$, which is a cofactor in the expression for the function $\mathcal{F}(\kappa, \mathrm{Fo})$ and depends only on Fo, can be used (Fig. 3). Using the curve mentioned, we can construct the dependences $F(\kappa, \mathrm{Fo})$ for the given r_0 and ρ^{s}.

We note that the analytical expression (17), which defines the relative contribution of radiation to the source temperature, is obtained under the following assumptions: 1) the temperature field is localized in an optically thin layer; 2) the contribution of radiation to heat transfer is small compared to the molecular (conductive) mechanism of heat transfer; 3) reflection at the boundary is close to mirror reflection ($\rho^{s} \approx 1$). A physical model of measurement corresponded to its idealization: a linear source of small radius (a filament) placed in an infinite medium (a liquid) was heated by rectangular pulses of current. The heating of the filament is small ($\Delta T \approx 1$ K), which allows a linear approximation of the radiative heat flux.

It is of interest to compare relation (17), which defines the relative contribution of radiation, with a similar relation from [3], which, as has already been mentioned, fails to allow for radiation from the surface of the heater. We represent the latter expression in a form similar to (17)

$$\frac{\delta T_{\mathrm{rad}}^{[3]}}{\Delta T_{\mathrm{cond}}} = \frac{2n^2 \sigma T_0^3 \pi^2 r_0^2}{\lambda \mathcal{F}_{\mathrm{cond}}(\mathrm{Fo})} \left[\frac{\kappa}{4} \left(\ln \frac{4\mathrm{Fo}}{\exp(C_{\mathrm{E}})} + 1 \right) - \kappa \, \mathrm{Fo} \right] = \frac{2n^2 \sigma T_0^3 \pi^2 r_0^2}{\lambda \mathcal{F}_{\mathrm{cond}}(\mathrm{Fo})} \mathcal{F}_{\ln}(\kappa, \mathrm{Fo}), \tag{18}$$

where

$$\mathcal{F}_{\ln}(\kappa, \mathrm{Fo}) = \frac{\kappa}{4} \left(\ln \frac{4\mathrm{Fo}}{\exp(C_{\mathrm{E}})} + 1 \right) - \kappa \, \mathrm{Fo}.$$

Omitting the same coefficients in (17) and (18), we compare $\mathcal{F}(\kappa, \mathrm{Fo})$ and $\mathcal{F}_{\ln}(\kappa, \mathrm{Fo})$ (Fig. 4). It is seen from the graphs that for weakly absorbing media ($\kappa \sim 10$–$100\ \mathrm{m}^{-1}$), in studying which a method of short-

TABLE 1. Thermal Conductivity of Fluorocarbons on the Saturation Line

Liquid	Temperature range, K	Smoothing polynomial λ, mW/(m·K)
Perfluorooctane, $C_{18}F_{18}$	263–463	$65.9-1.33\cdot10^{-1}(T-273)-1.2\cdot10^{-5}(T-273)^2$
Perfluorodecalin, $C_{10}F_{18}$	253–473	$60.1-0.105(T-273)$
Perfluoro-4, 7, 10, 13-tetraoxa-5, 8, 9, 12-tetramethylhexadecane, $C_{16}F_{34}O_4$	213–530	$64.9-0.051(T-273)$
Perfluoro-4, 7, 10-trioxa-5, 8, 9-trimethyltridecane, $C_{13}F_{28}O_3$	263–493	$67.0-0.07(T-273)$
Perfluoro-2, 4, 6, 8, 10, 12, 15, 17, 19, 21, 23, 25-dodecaoxahexacosane, $C_{14}F_{30}O_{12}$	213–503	$86.8-0.010(T-273)-0.361\cdot10^{-3}(T-273)^2$

duration measurements is most efficient, calculation by formula (18) strongly deviates from the more accurate result of (17).

We evaluate the effect of radiation on the results of measuring the molecular thermal conductivity of organic liquids using relation (17).

Assuming, for definiteness, that an optically thin layer is realized in the experiments at $\mathrm{Kn} = \kappa^*\sqrt{at} = 0.1$ and the measurement time is $t = 5\cdot10^{-2}$ sec, we can obtain the limiting value of the coefficient of absorption κ^*. Assuming $a \sim 10^{-8}$ m^2/sec, we have the estimate $\kappa^* = 0.5\cdot10^4$ m^{-1}.

For liquids with $\kappa < \kappa^*$, calculation by (14) is correct. Actually, the range mentioned determines the entire class of weakly absorbing liquids, i.e., media where radiative transfer is pronounced.

Using the limiting value of κ^* and (17), we can obtain the following important result: in short-duration measurements ($t = 5\cdot10^{-2}$ sec) of the thermal conductivity of semitransparent organic liquids by a nonstationary method of hot filament (the filament radius is $\sim10^{-6}$ m) the measurement error for the molecular thermal conductivity (for temperatures lower than 500 K) due to neglect of radiation does not exceed 0.1%.

Table 1 presents the results of a study of the thermal conductivity of five fluorocarbons made according to the technique of [8]. From the viewpoint of radiative heat transfer, the studied liquids satisfy the condition $\kappa < \kappa^*$ and, correspondingly, the measurement results can be classified unambiguously as measurements of molecular thermal conductivity. The measurement error for the thermal conductivity did not exceed 1.5%; the effect of radiation was no higher than 0.1%.

It should be noted that in using stationary methods, due to the relatively small values of the molecular thermal conductivity of fluorocarbons, the effect of radiation can be appreciable. For example, when a stationary method of plane layer is used, a liquid layer of thickness 1 mm with a coefficient of absorption $\kappa = \kappa^* = 5\cdot10^4$ m^{-1} can be considered to be optically thick. In this case, the Rosseland relative component of the effective thermal conductivity could amount to 1% at 100°C and 4% at 300°C.

NOTATION

T, temperature, K; a, thermal diffusivity, m^2/sec; q_{lin}, heat flux per unit length of the source, W/m; r, coordinate, m; t, time, sec; λ, thermal conductivity, W/(m·K); q^{rad}, radiative heat flux, W/m^2; ρ, density, kg/m^3; c, specific heat, J/(kg·K); r_0, source radius, m; κ, coefficient of absorption, m^{-1}; n, dimensionless refraction index; T_0, reference temperature, K; Kn, dimensionless Knudsen number; $\mathrm{Fo} = at/r_0^2$, dimensionless Fourier number; $\sigma = 5.67\cdot10^{-8}$ J/(m^2·sec·K^4), Stefan–Boltzmann constant; ρ^s, dimensionless coefficient of mirror reflection; Y_n, J_n, Bessel function [9]; L, Laplace transform operator; K_m, modified Bessel function of the second kind; s, Laplace variable; $C_E = 0.577$, Euler constant; θ, Laplace transform of the function T: $T(r, t) \xrightarrow{L} \theta(r, s)$; T_{ln}, temperature expressed by a logarithmic function (by formula (15)); δT_{rad}, radiation contribution to the temperature (the last term in formula (14)). Subscripts: lin, linear; 0, initial value; cond, conductive.

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